DERIVACE FUNKCE

Vzorce pro derivování:

1) $(c \cdot f)' = c \cdot f'$, $c \in \mathbb{R}$

2) $(f \pm g)' = f' \pm g'$

3) $(f \cdot g)' = f' \cdot g + f \cdot g'$

4) $\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$

Derivace elementárních funkcí

$f(x)$	$f'(x)$
c	0
x^n	$n \cdot x^{n-1}$
e^x	e^x
a^x	$a^x \cdot \ln a$
$\ln x$	$\frac{1}{x}$
$\log_a x$	$\frac{1}{x \cdot \ln a}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\operatorname{tg} x$	$\frac{1}{\cos^2 x}$
$\operatorname{cotg} x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\operatorname{arctg} x$	$\frac{1}{1+x^2}$
$\operatorname{arccotg} x$	$-\frac{1}{1+x^2}$

$\rightarrow$ $\underline{\underline{(x)'}} = (x^1)' = 1 \cdot x^{1-1} = 1 \cdot x^0 = 1 \cdot 1 = \underline{\underline{1}}$

Př: Derivujte funkce a výsledek upravte.

1) $f(x) = x^7 - 2x^5 + 3x - 4 + \operatorname{cotg} x + \arcsin x$

$$f'(x) = (x^7)' - 2\cdot(x^5)' + 3\cdot(x)' - (4)' + (\operatorname{cotg} x)' + (\arcsin x)' =$$
$$= 7\cdot x^{7-1} - 2\cdot 5\cdot x^{5-1} + 3\cdot 1 - 0 + \left(-\frac{1}{\sin^2 x}\right) + \frac{1}{\sqrt{1-x^2}} =$$
$$= 7x^6 - 10x^4 + 3 - \frac{1}{\sin^2 x} + \frac{1}{\sqrt{1-x^2}}$$

2) $f(x) = 3\operatorname{tg} x\cdot(1-\operatorname{arctg} x)$ ⇒ derivace součinu funkcí (vzorec 3)

$$f'(x) = (3\operatorname{tg} x)'\cdot(1-\operatorname{arctg} x) + 3\operatorname{tg} x\cdot(1-\operatorname{arctg} x)' =$$
$$= 3\cdot\frac{1}{\cos^2 x}\cdot(1-\operatorname{arctg} x) + 3\operatorname{tg} x\cdot\left(0 - \frac{1}{1+x^2}\right) = \frac{3(1-\operatorname{arctg} x)}{\cos^2 x} - \frac{3\operatorname{tg} x}{1+x^2}$$

3) $f(x) = \dfrac{\cos x}{e^x}$ → derivace podílu funkcí (vzorec 4)

$$f'(x) = \frac{(\cos x)'\cdot e^x - \cos x\cdot(e^x)'}{(e^x)^2} = \frac{-\sin x\cdot e^x - \cos x\cdot e^x}{e^{2x}} = \frac{-e^x(\sin x + \cos x)}{e^{2x}} =$$
$$= -\frac{\sin x + \cos x}{e^x}$$

4) $f(x) = \dfrac{1-x^4}{\sqrt[3]{\pi}} = \underbrace{\dfrac{1}{\sqrt[3]{\pi}}}_{\text{konstanta}}\cdot(1-x^4)$

$$f'(x) = \frac{1}{\sqrt[3]{\pi}}\cdot(1-x^4)' = \frac{1}{\sqrt[3]{\pi}}\cdot(0-4x^3) = -\frac{4x^3}{\sqrt[3]{\pi}}$$

5) $f(x) = \underbrace{\dfrac{5x^2}{\sqrt[5]{x^2}}}_{5x^2\cdot x^{-\frac{2}{5}} = 5x^{2-\frac{2}{5}} = 5x^{\frac{8}{5}}} + 30\sqrt[15]{x} + \dfrac{6}{\sqrt[3]{x}} = 5x^{\frac{8}{5}} + 30x^{\frac{1}{15}} + 6x^{-\frac{1}{3}}$

$$f'(x) = 5\cdot\frac{8}{5}x^{\frac{8}{5}-1} + 30\cdot\frac{1}{15}\cdot x^{\frac{1}{15}-1} + 6\cdot\left(-\frac{1}{3}\right)\cdot x^{-\frac{1}{3}-1} = 8x^{\frac{3}{5}} + 2x^{-\frac{14}{15}} - 2x^{-\frac{4}{3}} =$$
$$= 8\sqrt[5]{x^3} + \frac{2}{\sqrt[15]{x^{14}}} - \frac{2}{\sqrt[3]{x^4}}$$

Sami derivujte:

6) $f(x) = x\ln x - x + \ln 3$

7) $f(x) = \dfrac{e^x-1}{e^x+1}$

8) $f(x) = x\sin x + \cos x$

9) $f(x) = \frac{1}{2}(x - \sin x\cdot\cos x)$

10) $f(x) = \operatorname{tg}\frac{\pi}{4} + \dfrac{\sin x}{1-\cos x}$

11) $f(x) = \dfrac{x\ln x}{1-x}$

6) $f(x) = x\ln x - x + \ln 3$ konstanta

$$f'(x) = \underbrace{1\cdot\ln x + x\cdot\frac{1}{x}}_{\text{derivace součinu funkcí}} - 1 + 0 = \ln x + 1 - 1 = \ln x$$

7) $f(x) = \frac{e^x - 1}{e^x + 1}$ → derivace podílu funkcí

$$f'(x) = \frac{(e^x - 0)(e^x+1) - (e^x - 1)(e^x + 0)}{(e^x+1)^2} = \frac{e^x(e^x+1) - (e^x-1)e^x}{(e^x+1)^2} =$$

$$= \frac{e^x(e^x + 1 - e^x + 1)}{(e^x+1)^2} = \frac{2e^x}{(e^x+1)^2}$$

8) $f(x) = x\sin x + \cos x$

$$f'(x) = \underbrace{1\cdot\sin x + x\cos x}_{\text{derivace součinu funkcí}} - \sin x = x\cos x$$

9) $f(x) = \frac{1}{2}(x - \sin x\cdot\cos x)$

$$f'(x) = \frac{1}{2}(1 - \underbrace{(\cos x\cdot\cos x + \sin x\cdot(-\sin x))}_{\text{derivace součinu funkcí}}) = \frac{1}{2}(\overbrace{1 - \cos^2 x}^{\sin^2 x} + \sin^2 x) =$$

$$= \frac{1}{2}\cdot 2\sin^2 x = \sin^2 x$$

10) $f(x) = \underbrace{\operatorname{tg}\frac{\pi}{4}}_{\text{konstanta}} + \frac{\sin x}{1 - \cos x}$ → derivace podílu funkcí

$$f'(x) = 0 + \frac{\cos x\cdot(1-\cos x) - \sin x\cdot\sin x}{(1-\cos x)^2} = \frac{\cos x - \cos^2 x - \sin^2 x}{(1-\cos x)^2} = \frac{\cos x - (\overbrace{\sin^2 x + \cos^2 x}^{1})}{(1-\cos x)^2} =$$

$$= \frac{\cos x - 1}{(\cos x - 1)^2} = \frac{1}{\cos x - 1}$$

11) $f(x) = \frac{x\ln x}{1-x}$ → derivace součinu funkcí → derivace podílu funkcí

$$f'(x) = \frac{(1\cdot\ln x + x\cdot\frac{1}{x})\cdot(1-x) - x\ln x\cdot(-1)}{(1-x)^2} = \frac{(\ln x + 1)(1-x) + x\ln x}{(1-x)^2} =$$

$$= \frac{\ln x - x\ln x + 1 - x + x\ln x}{(1-x)^2} = \frac{1 - x + \ln x}{(1-x)^2}$$